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SHORT NOTE

Direction of shear

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Abstract—The direction of maximum shear stress on any plane can be determined on a normal stereogram from the orientations and relative magnitudes of the principal stresses, without having to identify or evaluate the traction vector and without considering the stress ellipsoid. An analogous procedure is available for determining the direction of maximum strain or maximum strain rate across a plane.

INTRODUCTION

IN SOME applications, the magnitude of shear stress is of importance, for instance in microstructural work. In others, the ratio of magnitudes of shear and normal stresses may be important, for example in considerations of frictional stability. For such cases, a number of methods for evaluating both the *direction* and *magnitude* of the shear and normal stress components on a general plane have recently been described. However, there are applications in which the constraints on stress magnitudes are very poor. One may wish, for example, to consider whether a postulated paleostress regime is appropriate to the generation of striae in a given direction on a reactivated fault, despite minimal knowledge of the frictional parameters. In such cases it can be appropriate to determine solely the *direction* of maximum shear stress on a plane by a method which bypasses some of the steps necessary to the evaluation of magnitudes. This paper presents a method which requires neither a determination of the traction on the plane (cf. De Paor 1990, Ragan 1990) nor of the V angle of the Cauchy ellipsoid (Lisle 1989) as an intermediate stage. It involves less evaluation of trigonometrical functions and a simpler stereographic construction than does the method of Means (1989), to which it is most closely related.

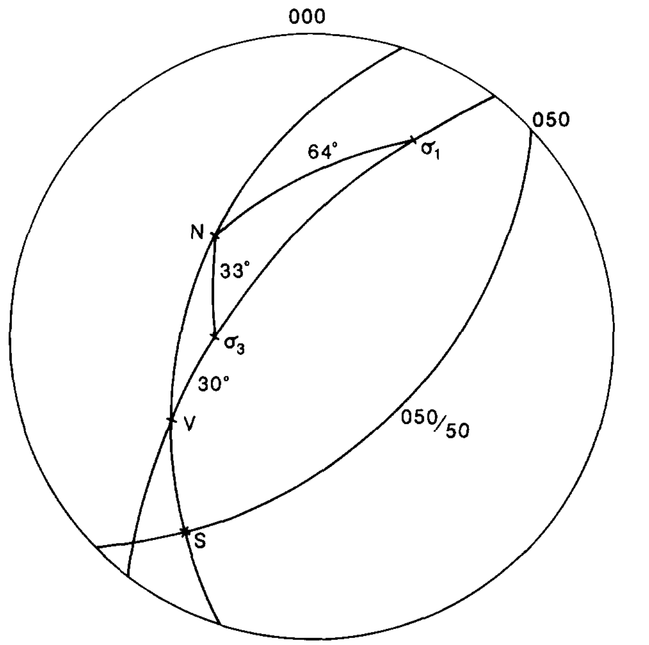

Fig. 1. An example of a stereogram (equal-angle, lower-hemisphere) to illustrate the procedure, for the following data: sample plane strike 050°, dip 50°SE. Principal stress orientations: σ_1 azimuth 030°, plunge 18°; σ_3 azimuth 274°, plunge 55°. Principal stress values: $\sigma_1 = 400$; $\sigma_2 = 300$; $\sigma_3 = 210$. In this case, the angle $\nu = -60°$ derived from these values would give a direction of V in the upper hemisphere; so $\nu = +120°$ $(=-60° + 180°)$ is used instead to plot the opposite direction along V at 120° from σ_1 (30° beyond σ_3). From this the great circle through N and V is constructed, giving an intersection with the great circle of the sample plane at S, the direction of maximum shear stress on the sample plane.

PROCEDURE

Step 1. Draw the great circle representing the orientation of the sample plane and plot the directions of its normal, N, and of the σ_1 and σ_3 principal stress axes on a stereogram (of either hemisphere), as in Fig. 1.

Step 2. Measure the angles between N and the σ_1 and σ_3 axes, and evaluate $l = \cos(N \wedge \sigma_1)$ and $n = \cos(N \wedge \sigma_3)$, if these are not known.

Step 3. Evaluate the angle $\nu = \arctan[((\sigma_3 - \sigma_2)n)/((\sigma_1 - \sigma_2)l)]$.

Step 4. Plot the direction V, which lies in the plane of σ_1 and σ_3, at an angle of ν from the plotted σ_1 direction (positive ν towards plotted σ_3, negative ν away from plotted σ_3). If this direction is away from the plotted hemisphere, add 180° to ν first, to obtain the direction along V which falls within the plot.

Step 5. Draw the great circle containing N and V. This intersects the circle of the sample plane at its direction of maximum shear stress, S.

To determine the sense of shear, identify, for each end of S, on which side of the sample plane it lies within 90° of the σ_1 axis and on which it lies within 90° of the σ_3 axis. On each side of the plane, the shear stress acts from the end of S within 90° of the direction of maximum compression (minimum tension) towards the end within 90° of the direction of minimum compression (maximum tension).

PROOF IN BRIEF

A plane with unit normal $\mathbf{n}$ experiences traction $\mathbf{t} = \boldsymbol{\sigma}\mathbf{n}$. Taking the principal stress directions as coordinate axes, so that tensor $\boldsymbol{\sigma}$ is diagonalized, a vector $\mathbf{v} = \mathbf{t} - \sigma_2\,\mathbf{n}$ can be seen to have a component of zero magnitude in the direction of σ_2 and therefore to lie in the plane of σ_1 and σ_3. The angle at which $\mathbf{v}$ lies between these principal axes is given by the arctangent of $((\sigma_3 - \sigma_2)n_3)/((\sigma_1 - \sigma_2)n_1)$, the ratio of its components along them. All vectors, including both $\mathbf{v}$ and the shear component vector $\mathbf{s}$, of the form $\mathbf{t} \pm c\mathbf{n}$ (c real) lie in a plane represented by the great circle plotted through the directions of any convenient pair of such vectors ($\mathbf{n}$ and $\mathbf{v}$ in our case). S (along $\mathbf{s}$) is the intersection of this plane with the plane for which the shear direction is being determined.

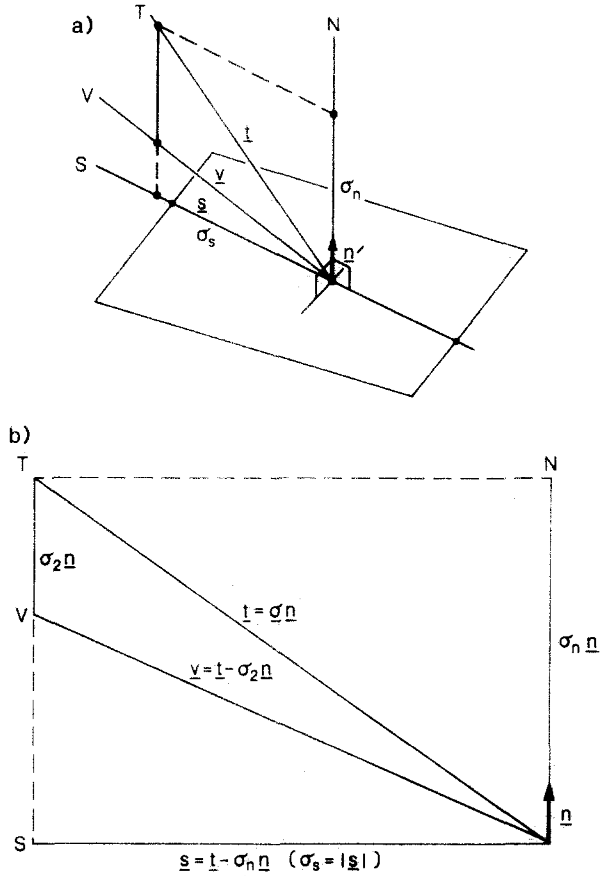

Fig. 2. The plane of traction on a sample plane. Directions, labelled as in the text by capital letters, are the normal, N, the traction, T, the intersection with the σ_1, σ_3 principal plane, V, and the shear along the sample plane, S. Their corresponding vectors are vector components of the traction with the exception of the unit normal $\mathbf{n}$ to the sample. (a) A perspective view, drawn with the plane of the sample horizontal. (b) The geometric relationships between vector magnitudes within the traction plane. Underlined letters indicate vectors.

DETAILED EXPLANATION

A more detailed explanation in terms of direction cosines and components may be more intelligible than the above. A unit vector may be specified by direction cosines (l, m, n) which are the cosines of its angles to three orthogonal reference axes and are also the magnitudes of its vector components in the directions of these axes. We will take as reference axes the directions of the principal stresses, and take the direction of each axis lying within the hemisphere of our stereogram to be in a positive sense. Let the unit vector in the direction N, normal to the sample plane and into the hemisphere of the stereogram, be $(l, m, n)^T$. The traction vector on the plane is $(\sigma_1 l, \sigma_2 m, \sigma_3 n)^T$. Its direction, which we will not bother to determine, we designate T.

Any vector which is the sum or difference of these two vectors, or any multiple of them, will lie in the NT plane. One such vector of particular interest is that having zero magnitude for its component in the σ_2 direction. This vector is obtained by subtracting from the traction vector one of magnitude σ_2 along the plane normal, N. Its components are

$$\begin{pmatrix} (\sigma_1 - \sigma_2)l \\ 0 \\ (\sigma_3 - \sigma_2)n \end{pmatrix} = \begin{pmatrix} \sigma_1 l \\ \sigma_2 m \\ \sigma_3 n \end{pmatrix} - \begin{pmatrix} \sigma_2 l \\ \sigma_2 m \\ \sigma_2 n \end{pmatrix}.$$

Its direction, V, lies both within the NT plane and the principal plane containing σ_1 and σ_3; it is their direction of intersection. Its orientation within the $\sigma_1\sigma_3$ principal plane (given as angle v above) can be obtained from the ratio of its components in these directions, which are $(\sigma_1 - \sigma_2)l$ along the σ_1 axis and $(\sigma_3 - \sigma_2)n$ along the σ_3 axis.

The shear stress on the sample plane is the vector difference (Fig. 2) between the traction vector along T and a different vector (representing the normal stress component) along the plane normal, N. So its direction, S, also lies in the same plane as N, T and V. S also lies in the sample plane. Therefore it will plot on the stereogram at the intersection of the great circle drawn through N and V with that representing the sample plane.

COMMENT

Apart from its practical simplicity, this procedure has didactic use in developing an understanding of stress. It highlights the distinction between two different three-dimensional geometries which we use simultaneously when considering stress:

(1) a geometric representation of the stress state, which has symmetry about its principal planes; and

(2) the geometry of traction on a generally oriented surface. The plane of symmetry of the latter (Fig. 2) contains the direction of the traction vector, its component (the normal stress) normal to the sample surface and its component along the sample surface (the shear stress). This method determines the direction V of intersection of the traction plane with the $\sigma_1\sigma_3$ principal plane. In doing so it requires that the conceptual distinction be made between these two geometries, and establishes that their planes of symmetry are generally not coincident.

This method, as described, takes no account of the polarities of the vectors and axes determined. It would be possible to follow these step by step through the procedure in order to deduce the sense of the shear along S. Practically it is far simpler to determine the S direction first and then use an independent test of shear sense, as suggested above.

STRAIN

The reference state for this discussion of strain is *the strained state*, in which the orientations of any sample plane and of the principal strain axes might be determined without any unstraining procedure. For determination of the direction *in a sample plane* of the maximum shear strain *across* the plane, the *directions* of the principal stretches, S_1, S_2, S_3, in the strained state take the place, as reference axes, of those of the principal stresses, $\sigma_1, \sigma_2, \sigma_3$, in the above procedure.

The effect of the stretch (the irrotational component of strain) on the orientation of a sample plane is best considered in terms of its reciprocal space vector representation. The orientation of the sample plane in the strained state is the same as one which cuts the principal stretch axes at intercepts from the origin of $1/l$, $1/m$ and $1/n$, where l, m and n are the direction cosines of its unit normal vector $\mathbf{n}$. Removal of the stretch ('unstraining') gives intercepts of $1/S_1l$, $1/S_2m$ and $1/S_3n$, for the orientation of the sample plane in its unstretched state. The direction of the normal to the sample plane in the unstretched state is given by the vector (not of unit magnitude) $(S_1l, S_2m, S_3n)^{\mathrm{T}} = \mathbf{Sn}$. (Note that the operation of $\mathbf{S}$ on the reciprocal space vector, representing the orientation of a *plane* in real space, corresponds to the effect of the reciprocal of $\mathbf{S}$ in real space.)

The pre-stretch material line along the above vector $\mathbf{Sn}$ has been transformed by the stretch to the material line vector

$$\mathbf{m} = \mathbf{SSn} = \begin{pmatrix} S_1{}^2l \\ S_2{}^2m \\ S_3{}^2n \end{pmatrix} = \begin{pmatrix} \lambda_1 l \\ \lambda_2 m \\ \lambda_3 n \end{pmatrix} = \boldsymbol{\lambda}\mathbf{n},$$

where λ_1, λ_2 and λ_3, the principal quadratic stretches, are the components of the diagonalized quadratic stretch tensor, $\boldsymbol{\lambda}$. The shear across the sample plane, referred to the strained state, is the angle between its normal, represented by $\mathbf{n}$, and the direction of that material line which was normal before the strain, represented by $\mathbf{m}$. The trace of the plane containing $\mathbf{n}$ and $\mathbf{m} = \boldsymbol{\lambda}\mathbf{n}$ on the sample plane may be determined by the same procedure as described for $\mathbf{n}$ and $\mathbf{t} = \boldsymbol{\sigma}\mathbf{n}$ in the case of stress, with σ replaced throughout by λ. On each side of the sample plane, the sense of shear is towards the end of S within 90° of the direction of maximum stretch (the λ_1 or S_1 axis), from the end within 90° of the minimum stretch (λ_3 or S_3) axis.

STRAIN RATE

The analogous procedure for shear strain rate is simpler than that for shear strain; no finite rotation of the sample plane is involved. As each of the transformations $\mathbf{S}$ described above tends to infinitesimal, so $(\mathbf{m} - \mathbf{n})$ tends to the differential of $\boldsymbol{\lambda}\mathbf{n}$, which is $2\dot{\mathbf{S}}\mathbf{n}$. The coefficient 2 is of no consequence to the procedure used and may be neglected. The trace of the plane containing $\mathbf{n}$ and $\dot{\mathbf{S}}\mathbf{n}$ on the sample plane may be determined by the same procedure as described for $\mathbf{n}$ and $\boldsymbol{\sigma}\mathbf{n}$ in the case of stress, with σ replaced throughout by $\dot{S}$. On each side of the sample plane, the sense of shear is towards the end of S within 90° of the $\dot{S}_1$ axis, from the end within 90° of the $\dot{S}_3$ axis.

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